
Calcul de limites

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow 0} (x+1)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x} = +\infty$$

$$\lim_{x \rightarrow -\infty} e^x x = 0$$

$$\lim_{x \rightarrow 0} \frac{-1 + e^x}{x} = 1$$

$$\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} = 0$$

$$\lim_{x \rightarrow 0} x \ln(x) = 0$$

■ Exercices :

- utiliser la règle de l'Hospital si nécessaire

1) $\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x^3}$

2) $\lim_{x \rightarrow +\infty} e^x x^2$

3) $\lim_{x \rightarrow 0} (-e^{-x} + e^x) \frac{1}{\sin(x)}$

4) $\lim_{x \rightarrow 0} \frac{\ln\left(\frac{x+1}{1-x}\right)}{x}$

5) $\lim_{x \rightarrow 0} \ln(x) \operatorname{tg}(x)$

6) $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x$

7) $\lim_{x \rightarrow +\infty} \left(1 + \frac{3}{x}\right)^x$

8) $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{5x}$

9) $\lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x}\right)^{3x}$

10) $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{x+5}$

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11) $\lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x}\right)^{2-x}$

12) $\lim_{x \rightarrow +\infty} \left(\frac{x+2}{x-3}\right)^{3x-1}$

13) $\lim_{x \rightarrow 0} (1 - 3x)^{-2/x}$

14) $\lim_{x \rightarrow 0} (\sin(x) + 1)^{\frac{1}{\sin(x)}}$

15) $\lim_{x \rightarrow +\infty} \left(\frac{x+1}{2x+1}\right)^{x^2}$

16) $\lim_{x \rightarrow +\infty} \left(\frac{x-1}{x+1}\right)^x$

17) $\lim_{x \rightarrow +\infty} \left(\frac{1}{x^2}\right)^{\frac{2x}{x+1}}$

18) $\lim_{x \rightarrow +\infty} \left(\frac{x^2+2}{2x^2+1}\right)^{x^2}$

■ Solutions

- 1) $\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x^3} = 0$
- 2) $\lim_{x \rightarrow +\infty} e^x x^2 = +\infty$
- 3) $\lim_{x \rightarrow 0} (-e^{-x} + e^x) \frac{1}{\sin(x)} = 2$
- 4) $\lim_{x \rightarrow 0} \frac{\ln\left(\frac{x+1}{1-x}\right)}{x} = 2$
- 5) $\lim_{x \rightarrow 0} \ln(x) \operatorname{tg}(x) = 0$
- 6) $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = e$
- 7) $\lim_{x \rightarrow +\infty} \left(1 + \frac{3}{x}\right)^x = e^3$
- 8) $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{5x} = e^5$
- 9) $\lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x}\right)^{3x} = e^6$
- 10) $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{x+5} = e$
- 11) $\lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x}\right)^{2-x} = \frac{1}{e^2}$
- 12) $\lim_{x \rightarrow +\infty} \left(\frac{x+2}{x-3}\right)^{3x-1} = e^{15}$
- 13) $\lim_{x \rightarrow 0} (1 - 3x)^{-2/x} = e^6$
- 14) $\lim_{x \rightarrow 0} (\sin(x) + 1)^{\frac{1}{\sin(x)}} = e$
- 15) $\lim_{x \rightarrow +\infty} \left(\frac{x+1}{2x+1}\right)^{x^2} = 0$
- 16) $\lim_{x \rightarrow +\infty} \left(\frac{x-1}{x+1}\right)^x = \frac{1}{e^2}$
- 17) $\lim_{x \rightarrow +\infty} \left(\frac{1}{x^2}\right)^{\frac{2x}{x+1}} = 0$
- 18) $\lim_{x \rightarrow +\infty} \left(\frac{x^2+2}{2x^2+1}\right)^{x^2} = 0$

■ Détail:

$$15) \lim_{x \rightarrow +\infty} \left(\frac{x+1}{2x+1}\right)^{x^2} = \lim_{x \rightarrow +\infty} \left(\frac{2x+1-x}{2x+1}\right)^{x^2} = \lim_{x \rightarrow +\infty} \left(1 - \frac{x}{2x+1}\right)^{x^2} = \lim_{x \rightarrow +\infty} \left(1 - \frac{1}{2}\right)^{x^2} = \lim_{x \rightarrow +\infty} \frac{1}{2^{x^2}} = 0$$

$$17) \lim_{x \rightarrow +\infty} \left(\frac{1}{x^2}\right)^{\frac{2x}{x+1}} = \lim_{x \rightarrow +\infty} \left(\frac{1}{x^2}\right)^2 = 0$$

$$18) \lim_{x \rightarrow +\infty} \left(\frac{x^2+2}{2x^2+1}\right)^{x^2} = \lim_{x \rightarrow +\infty} \left(\frac{2x^2+1-x^2+1}{2x^2+1}\right)^{x^2} = \lim_{x \rightarrow +\infty} \left(1 + \frac{1-x^2}{2x^2+1}\right)^{x^2} = \lim_{x \rightarrow +\infty} \left(1 - \frac{1}{2}\right)^{x^2} = 0$$

on peut aussi poser $y = x^2$

■ Exercices :

Sachant que $\lim_{x \rightarrow +\infty} \left(\frac{k}{x} + 1 \right)^x = e^k$ calculer

$$1) \lim_{x \rightarrow 0} \left(\frac{x+2}{3-x} \right)^x$$

$$2) \lim_{x \rightarrow 1} \left(\frac{x-1}{x^2-1} \right)^{x+1}$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{1}{x^2} \right)^{\frac{2x}{x+1}}$$

$$4) \lim_{x \rightarrow 0} \left(\frac{x^2 - 2x + 3}{x^2 - 3x + 2} \right)^{\frac{\sin(x)}{x}}$$

$$5) \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 2}{2x^2 + 1} \right)^{x^2}$$

$$6) \lim_{m \rightarrow +\infty} \left(1 - \frac{1}{m} \right)^m$$

$$7) \lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x} \right)^x$$

$$8) \lim_{x \rightarrow +\infty} \left(\frac{x}{x+1} \right)^x$$

$$9) \lim_{x \rightarrow +\infty} \left(\frac{x+1}{x+3} \right)^{x+2}$$

$$10) \lim_{m \rightarrow +\infty} \left(\frac{x}{m} + 1 \right)^m$$

$$11) \lim_{x \rightarrow 0} (\sin(x) + 1)^{\frac{1}{x}}$$

Solutions

$$1) \lim_{x \rightarrow 0} \left(\frac{x+2}{3-x} \right)^x = 1$$

$$2) \lim_{x \rightarrow 1} \left(\frac{x-1}{x^2-1} \right)^{x+1} = \frac{1}{4}$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{1}{x^2} \right)^{\frac{2x}{x+1}} = 0$$

$$4) \lim_{x \rightarrow 0} \left(\frac{x^2 - 2x + 3}{x^2 - 3x + 2} \right)^{\frac{\sin(x)}{x}} = \frac{3}{2}$$

$$5) \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 2}{2x^2 + 1} \right)^{x^2} = 0$$

$$6) \lim_{m \rightarrow +\infty} \left(1 - \frac{1}{m} \right)^m = \frac{1}{e}$$

$$7) \lim_{x \rightarrow +\infty} \left(1 + \frac{2}{x} \right)^x = e^2$$

$$8) \lim_{x \rightarrow +\infty} \left(\frac{x}{x+1} \right)^x = \frac{1}{e}$$

$$9) \lim_{x \rightarrow +\infty} \left(\frac{x+1}{x+3} \right)^{x+2} = \frac{1}{e^2}$$

$$10) \lim_{m \rightarrow +\infty} \left(\frac{x}{m} + 1 \right)^m = e^x$$

$$11) \lim_{x \rightarrow 0} (\sin(x) + 1)^{\frac{1}{x}} = e$$