Domaine de définition

Le domaine de définition d'une fonction réelle f est l'ensemble

$\operatorname{dom} f = \{x \in \mathbb{R} : f(x) \in \mathbb{R}\}$

Déterminer le domaine de définition des fonctions suivantes:

1) $f(x) = \dfrac{16x^2 - 2x + 8}{x^2 + 5x + 6}$

2) $f(x) = \sqrt{x^2 + 3x - 10}$

3) $f(x) = \dfrac{x + 6}{x^3 + 5x}$

4) $f(x) = \sqrt{4x - x^3}$

5) $f(x) = \sqrt{3x - 2}$

6) $f(x) = \dfrac{8x^2 - 5x + 3}{x^2 - 5x + 6}$

7) $f(x) = \sqrt{x^2 - 3x - 18}$

8) $f(x) = \dfrac{4x^2 - 5x + 15}{x^3 + 6x}$

9) $f(x) = \dfrac{1}{\sqrt{2x - x^3}}$

10) $f(x) = \dfrac{\sqrt{2 - x}}{\sqrt{5x - 1}}$

11) $f(x) = \dfrac{\sqrt{x^2 - 3x + 2}}{\sqrt{2x - 1}}$

12) $f(x) = \dfrac{x^2 + x - 1}{\sqrt{2x^2 - 3x + 1}}$

13) $f(x) = \sqrt{x + 7} + \sqrt{2x^2 - 3x - 9}$

14) $f(x) = \sqrt{x + 4} + \sqrt{x^2 - 3x - 10}$

15) $f(x) = \dfrac{\sqrt{2x^2 - 5x - 3}}{x^2 - 2x - 3}$

16) $f(x) = \dfrac{x^2 + 5x - 7}{\sqrt{2x^2 + 3x - 2}}$

17) $f(x) = \dfrac{\sqrt{3 - x}}{\sqrt{4x - 1}}$

18) $f(x) = \dfrac{\sqrt{x^2 - 5}}{x + 1}$

19) $f(x) = \sqrt{x + 1} + \dfrac{1}{8 - x^3}$

20) $f(x) = \sqrt{\dfrac{2 - 5x}{x^2 - 6x + 5}}$

Solutions

1) $dom\ f = \mathbb{R} \setminus \{-3,-2\}$

2) $dom\ f = \longleftarrow; -5] \cup [2; \longrightarrow$

3) $dom\ f = \mathbb{R} \setminus \{0\}$

4) $dom\ f = \longleftarrow; -2] \cup [0; 2]$

5) $dom\ f = [\frac{2}{3}; \longrightarrow$

6) $dom\ f = \mathbb{R} \setminus \{2,3\}$

7) $dom\ f = \longleftarrow; -3] \cup [6; \longrightarrow$

8) $dom\ f = \mathbb{R} \setminus \{0\}$

9) $dom\ f = \longleftarrow; -\sqrt{2}\,[\cup]0; \sqrt{2}\,[$

10) $dom\ f =]\frac{1}{5}; 2]$

11) $dom\ f =]\frac{1}{2}; 1] \cup [2; \longrightarrow$

12) $dom\ f = \longleftarrow; \frac{1}{2}[\cup]1; \longrightarrow$

13) $dom\ f = [-7; -\frac{3}{2}] \cup [3; \longrightarrow$

14) $dom\ f = [-4; -2] \cup [5; \longrightarrow$

15) $dom\ f = \longleftarrow; -1[\cup]-1; -\frac{1}{2}] \cup]3; \longrightarrow$

16) $dom\ f = \longleftarrow; -2[\cup]\frac{1}{2}; \longrightarrow$

17) $dom\ f =]\frac{1}{4}; 3]$

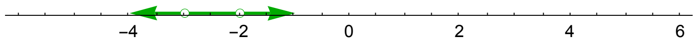
18) $dom\ f = \longleftarrow; -\sqrt{5}\,] \cup [\sqrt{5}\,; \longrightarrow$

19) $dom\ f = [-1; 2[\cup]2; \longrightarrow$

20) $dom\ f = \longleftarrow; \frac{2}{5}] \cup]1; 5[$

1) $f(x) = \dfrac{16x^2 - 2x + 8}{x^2 + 5x + 6}$

$x^2 + 5x + 6 \neq 0 \Leftrightarrow x \neq -3 \bigwedge x \neq -2 \Leftrightarrow x \in \mathbb{R} \setminus \{-3,-2\}$

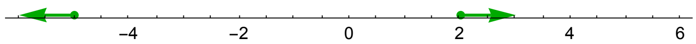


dom $f = \mathbb{R} \setminus \{-3,-2\}$

2) $f(x) = \sqrt{x^2 + 3x - 10}$

$x^2 + 3x - 10 \geq 0 \Leftrightarrow x \leq -5 \bigvee x \geq 2 \Leftrightarrow x \in \longleftarrow; -5] \bigcup [2; \longrightarrow$

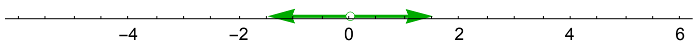
x		-5		2	
$x^2 + 3x - 10$	$+$	0	$-$	0	$+$



dom $f = \longleftarrow; -5] \bigcup [2; \longrightarrow$

3) $f(x) = \dfrac{x + 6}{x^3 + 5x}$

$x^3 + 5x \neq 0 \Leftrightarrow x \neq 0 \Leftrightarrow x \in \mathbb{R} \setminus \{0\}$

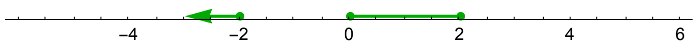


dom $f = \mathbb{R} \setminus \{0\}$

4) $f(x) = \sqrt{4x - x^3}$

$4x - x^3 \geq 0 \Leftrightarrow x \leq -2 \bigvee 0 \leq x \leq 2 \Leftrightarrow x \in \longleftarrow; -2] \bigcup [0; 2]$

x		-2		0		2	
$4x - x^3$	$+$	0	$-$	0	$+$	0	$-$

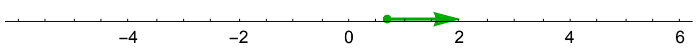


dom $f = \longleftarrow; -2] \bigcup [0; 2]$

5) $f(x) = \sqrt{3x - 2}$

$3x - 2 \geq 0 \Leftrightarrow x \geq \dfrac{2}{3} \Leftrightarrow x \in [\dfrac{2}{3}; \longrightarrow$

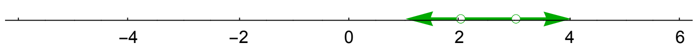
x		$\frac{2}{3}$	
$3x - 2$	$-$	0	$+$



dom $f = [\dfrac{2}{3}; \longrightarrow$

6) $f(x) = \dfrac{8x^2 - 5x + 3}{x^2 - 5x + 6}$

$x^2 - 5x + 6 \neq 0 \Leftrightarrow x \neq 2 \bigwedge x \neq 3 \Leftrightarrow x \in \mathbb{R} \setminus \{2,3\}$

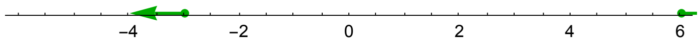


dom $f = \mathbb{R} \setminus \{2,3\}$

7) $f(x) = \sqrt{x^2 - 3x - 18}$

$x^2 - 3x - 18 \geq 0 \Leftrightarrow x \leq -3 \bigvee x \geq 6 \Leftrightarrow x \in \longleftarrow; -3] \bigcup [6; \longrightarrow$

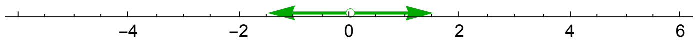
x		-3		6	
$x^2 - 3x - 18$	$+$	0	$-$	0	$+$



dom $f = \longleftarrow; -3] \bigcup [6; \longrightarrow$

8) $f(x) = \dfrac{4x^2 - 5x + 15}{x^3 + 6x}$

$x^3 + 6x \neq 0 \Leftrightarrow x \neq 0 \Leftrightarrow x \in \mathbb{R} \setminus \{0\}$

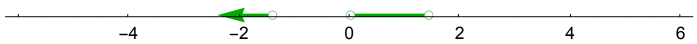


dom $f = \mathbb{R} \setminus \{0\}$

9) $f(x) = \dfrac{1}{\sqrt{2x - x^3}}$

$2x - x^3 > 0 \Leftrightarrow x < -\sqrt{2} \bigvee 0 < x < \sqrt{2} \Leftrightarrow x \in \longleftarrow; -\sqrt{2}\,[\bigcup]0; \sqrt{2}\,[$

x		$-\sqrt{2}$		0		$\sqrt{2}$	
$2x - x^3$	$+$	0	$-$	0	$+$	0	$-$



dom $f = \longleftarrow; -\sqrt{2}\,[\bigcup]0; \sqrt{2}\,[$

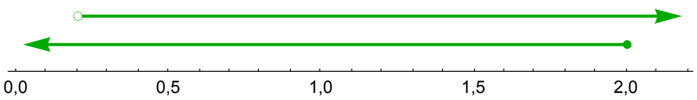
10) $f(x) = \dfrac{\sqrt{2 - x}}{\sqrt{5x - 1}}$

$2 - x \geq 0 \Leftrightarrow x \leq 2 \Leftrightarrow x \in \longleftarrow; 2]$

x		2	
$2 - x$	$+$	0	$-$

$5x - 1 > 0 \Leftrightarrow x > \dfrac{1}{5} \Leftrightarrow x \in \left]\dfrac{1}{5}; \longrightarrow\right.$

x		$\frac{1}{5}$	
$5x - 1$	$-$	0	$+$



dom $f = \left]\dfrac{1}{5}; 2\right]$

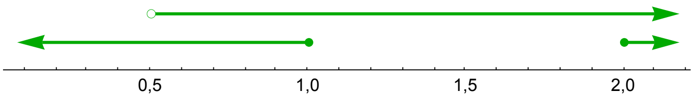
11) $f(x) = \dfrac{\sqrt{x^2 - 3x + 2}}{\sqrt{2x - 1}}$

$x^2 - 3x + 2 \geq 0 \Leftrightarrow x \leq 1 \bigvee x \geq 2 \Leftrightarrow x \in \longleftarrow; 1] \bigcup [2; \longrightarrow$

x		1		2	
$x^2 - 3x + 2$	$+$	0	$-$	0	$+$

$2x - 1 > 0 \Leftrightarrow x > \dfrac{1}{2} \Leftrightarrow x \in \left]\dfrac{1}{2}; \longrightarrow\right.$

x		$\frac{1}{2}$	
$2x-1$	$-$	0	$+$

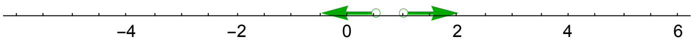


$\text{dom } f =]\frac{1}{2}; 1] \bigcup [2; \longrightarrow$

12) $f(x) = \dfrac{x^2+x-1}{\sqrt{2x^2-3x+1}}$

$2x^2-3x+1 > 0 \Leftrightarrow x < \frac{1}{2} \bigvee x > 1 \Leftrightarrow x \in \longleftarrow; \frac{1}{2}[\bigcup]1; \longrightarrow$

x		$\frac{1}{2}$		1	
$2x^2-3x+1$	$+$	0	$-$	0	$+$



$\text{dom } f = \longleftarrow; \frac{1}{2}[\bigcup]1; \longrightarrow$

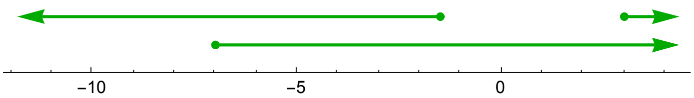
13) $f(x) = \sqrt{x+7} + \sqrt{2x^2-3x-9}$

$x+7 \geq 0 \Leftrightarrow x \geq -7 \Leftrightarrow x \in [-7; \longrightarrow$

x		-7	
$x+7$	$-$	0	$+$

$2x^2-3x-9 \geq 0 \Leftrightarrow x \leq -\frac{3}{2} \bigvee x \geq 3 \Leftrightarrow x \in \longleftarrow; -\frac{3}{2}] \bigcup [3; \longrightarrow$

x		$-\frac{3}{2}$		3	
$2x^2-3x-9$	$+$	0	$-$	0	$+$



$\text{dom } f = [-7; -\frac{3}{2}] \bigcup [3; \longrightarrow$

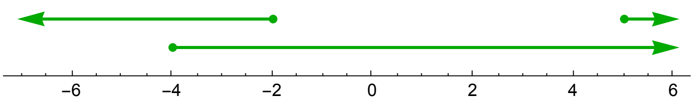
14) $f(x) = \sqrt{x+4} + \sqrt{x^2-3x-10}$

$x+4 \geq 0 \Leftrightarrow x \geq -4 \Leftrightarrow x \in [-4; \longrightarrow$

x		-4	
$x+4$	$-$	0	$+$

$x^2-3x-10 \geq 0 \Leftrightarrow x \leq -2 \bigvee x \geq 5 \Leftrightarrow x \in \longleftarrow; -2] \bigcup [5; \longrightarrow$

x		-2		5	
$x^2-3x-10$	$+$	0	$-$	0	$+$



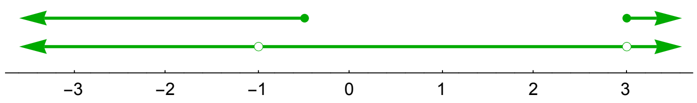
$\text{dom } f = [-4; -2] \bigcup [5; \longrightarrow$

15) $f(x) = \dfrac{\sqrt{2x^2 - 5x - 3}}{x^2 - 2x - 3}$

$x^2 - 2x - 3 \neq 0 \Leftrightarrow x \neq -1 \bigwedge x \neq 3 \Leftrightarrow x \in \mathbb{R} \setminus \{-1{,}3\}$

$2x^2 - 5x - 3 \geq 0 \Leftrightarrow x \leq -\dfrac{1}{2} \bigvee x \geq 3 \Leftrightarrow x \in \longleftarrow; -\dfrac{1}{2}] \bigcup [3; \longrightarrow$

x		$-\frac{1}{2}$		3	
$2x^2 - 5x - 3$	$+$	0	$-$	0	$+$

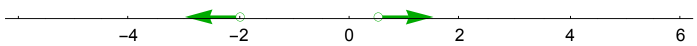


dom $f = \longleftarrow; -1[\bigcup]-1; -\dfrac{1}{2}] \bigcup]3; \longrightarrow$

16) $f(x) = \dfrac{x^2 + 5x - 7}{\sqrt{2x^2 + 3x - 2}}$

$2x^2 + 3x - 2 > 0 \Leftrightarrow x < -2 \bigvee x > \dfrac{1}{2} \Leftrightarrow x \in \longleftarrow; -2[\bigcup]\dfrac{1}{2}; \longrightarrow$

x		-2		$\frac{1}{2}$	
$2x^2 + 3x - 2$	$+$	0	$-$	0	$+$



dom $f = \longleftarrow; -2[\bigcup]\dfrac{1}{2}; \longrightarrow$

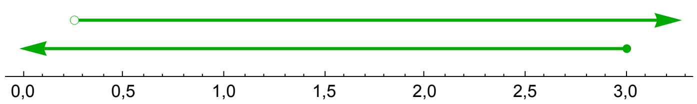
17) $f(x) = \dfrac{\sqrt{3 - x}}{\sqrt{4x - 1}}$

$3 - x \geq 0 \Leftrightarrow x \leq 3 \Leftrightarrow x \in \longleftarrow; 3]$

x		3	
$3 - x$	$+$	0	$-$

$4x - 1 > 0 \Leftrightarrow x > \dfrac{1}{4} \Leftrightarrow x \in]\dfrac{1}{4}; \longrightarrow$

x		$\frac{1}{4}$	
$4x - 1$	$-$	0	$+$



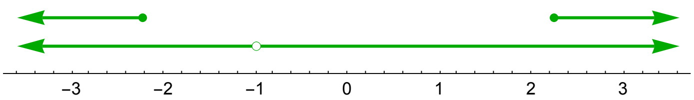
dom $f =]\dfrac{1}{4}; 3]$

18) $f(x) = \dfrac{\sqrt{x^2 - 5}}{x + 1}$

$x + 1 \neq 0 \Leftrightarrow x \neq -1 \Leftrightarrow x \in \mathbb{R} \setminus \{-1\}$

$x^2 - 5 \geq 0 \Leftrightarrow x \leq -\sqrt{5} \bigvee x \geq \sqrt{5} \Leftrightarrow x \in \longleftarrow; -\sqrt{5}] \bigcup [\sqrt{5}; \longrightarrow$

x		$-\sqrt{5}$		$\sqrt{5}$	
x^2-5	$+$	0	$-$	0	$+$



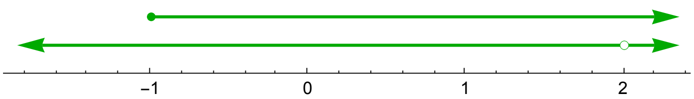
dom $f = \longleftarrow; -\sqrt{5}\,] \cup [\sqrt{5}\,; \longrightarrow$

19) $f(x) = \sqrt{x+1} + \dfrac{1}{8-x^3}$

$8-x^3 \neq 0 \Leftrightarrow x \neq 2 \Leftrightarrow x \in \mathbb{R} \setminus \{2\}$

$x+1 \geq 0 \Leftrightarrow x \geq -1 \Leftrightarrow x \in [-1; \longrightarrow$

x		-1	
$x+1$	$-$	0	$+$



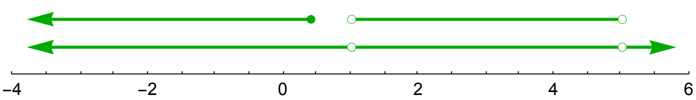
dom $f = [-1; 2[\cup]2; \longrightarrow$

20) $f(x) = \sqrt{\dfrac{2-5x}{x^2-6x+5}}$

$x^2-6x+5 \neq 0 \Leftrightarrow x \neq 1 \wedge x \neq 5 \Leftrightarrow x \in \mathbb{R} \setminus \{1{,}5\}$

$\dfrac{2-5x}{x^2-6x+5} \geq 0 \Leftrightarrow x \leq \dfrac{2}{5} \bigvee 1 < x < 5 \Leftrightarrow x \in \longleftarrow; \dfrac{2}{5}] \cup]1; 5[$

x		$\frac{2}{5}$		1		5	
$2-5x$	$+$	0	$-$	$-$	$-$	$-$	$-$
x^2-6x+5	$+$	$+$	$+$	0	$-$	0	$+$
$\frac{2-5x}{x^2-6x+5}$	$+$	0	$-$	$\vert$	$+$	$\vert$	$-$



dom $f = \longleftarrow; \dfrac{2}{5}] \cup]1; 5[$