

$$f(x) = \frac{\sqrt{x^2 + 1}}{x}$$

1. Domaine de definition

$$\text{Dom } f = \mathbb{R} \setminus \{0\}$$

2. Signe de f

x		0	
$\frac{\sqrt{x^2+1}}{x}$	-		+

3. Limites et asymptotes

$\lim_{x \rightarrow 0^-} \frac{\sqrt{x^2+1}}{x} = -\infty$
$\lim_{x \rightarrow 0^+} \frac{\sqrt{x^2+1}}{x} = \infty$

$$\text{AV} \equiv x = 0$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1}}{x} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x} = -1$$

$$\text{AH} \equiv x = 1 \text{ a droite}$$

$$\text{AH} \equiv x = -1 \text{ a gauche}$$

4. Intersection avec les axes

$$\text{Gf} \cap X = \{ \}$$

$$\text{Gf} \cap Y = \{ \}$$

5. Etude de f'

$$f'(x) = -\frac{1}{x^2 \sqrt{x^2 + 1}}$$

x		0	
$-\frac{1}{x^2 \sqrt{x^2+1}}$	-		-

6. Etude de f''

$$f''(x) = \frac{3x^2 + 2}{x^3 (x^2 + 1)^{3/2}}$$

x		0	
$\frac{3x^2+2}{x^3(x^2+1)^{3/2}}$	-		+

7. Tableau recapitulatif

x	$-\infty$		0		∞
f(x)	-1	-		+	1
	x = -1				x = 1
pente	0	-		-	0
concavite	0	-		+	0

8. Graphe de f

